Relationship of Shear Force and Punching Analysis of Reinforced Concrete Slabs

D. A. Pekin

1 Introduction

The calculation of reinforced concrete slabs for shear forces and punching was dealt with by leading Russian specialists both in the Soviet period and at the present time. Taking into account the actual sequence of the development of the theory of reinforced concrete, methods of calculation and implementation of structural solutions, the initial attention was mainly paid to the rod elements. In this regard, and also due to the relatively small volume of construction from monolithic reinforced concrete until the beginning of the 90s of the last century, for the calculation of slabs for the action of shear forces, assumptions and a methodology based on the rod analogy were used or only the calculation for punching was performed [1].

2 Relationship of Shear Force and Punching Failure

At Russian Research, Design and Technological Institute of Concrete and Reinforced Concrete, special experimental studies were carried out [2] to compare the mechanisms of failure along a sloping section and from punching, as well as to search for transitional forms that allow developing criteria for applying the calculation method for the action of shear forces or punching. The prototypes were tested, supported on two sides on a span of 1.4 m, 300 mm high, with a loading platform of 200 × 200 mm in the middle of the span (i.e., the sloping section span was 0.6 m), while varying their width in multiples of the size of the loading platform—200 (ПІ-1a), 3 × 200 (ПІ-2a), 5 × 200 (ПІ-3a) and 7 × 200 (ПІ-4a) mm, as well as control prototype with dimensions 1.4 × 1.4 × 0.3 (h) m (ПІ-5a), supported on 4 sides, to implement

D. A. Pekin (✉)
Moscow State University of Civil Engineering, Moscow, Russia

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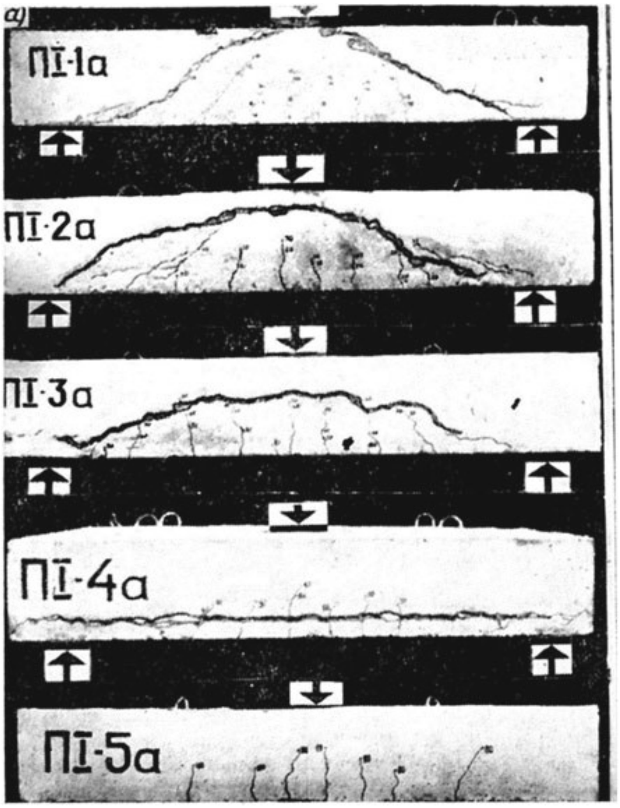

Fig. 1 Features of prototypes failure

standard punching mechanism (the marking of the prototypes, indicated in brackets, is shown in the photographs in Fig. 1).

Based on the results of this research work, the important issue of the similarity of fracture mechanisms in shear force and from punching was not considered, despite the results obtained during the experiments:

1. The formation and development of main sloping cracks was ignored in prototypes with dimensions close to square in plan, in which the punching mechanism was realized;
2. There were no proposals to improve the punching shear design method, which then and now does not take into account the formation and development of the main sloping cracks;
3. The effect of bending of the prototypes and the change in the height of the compressed zone of concrete both on the lateral faces of the prototypes and along the width of the cross section with an increase in its width were not taken into account (the height of the compressed zone of concrete along the width of the prototypes changes, increasing from the middle of the cross section to the lateral edges).

The following figure shows photographs of prototypes after failure, demonstrating the features of the formation and development of normal and sloping cracks on the

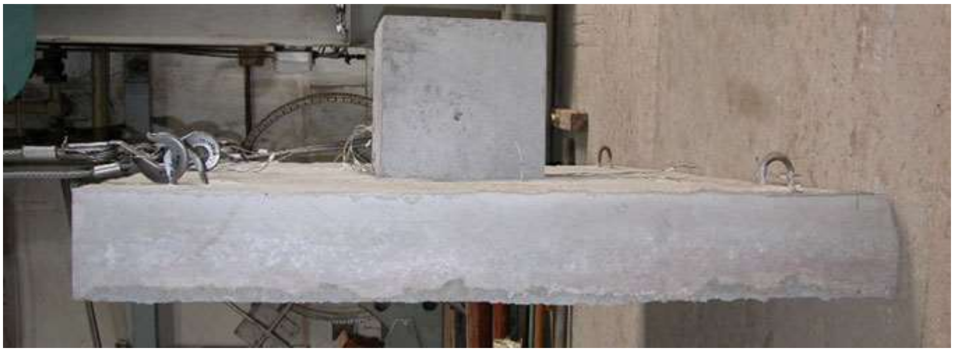

Fig. 2 Prototype in [7] after punching failure. Side view

free lateral faces of elements, with the exception of sample ПI-5a, which had a support.

A contradiction in the picture of the formation and development of cracks on the lateral face for prototype ПI-5a, shown in Fig. 1, which in the description [2] is indicated as supported on four sides, but at the same time having developed normal cracks on the lateral surface, characteristic of the support of the prototype only on two sides (when supported along the contour, such normal cracks on the lateral faces cannot form, especially along mid-span, see Fig. 2). It is possible that the prototype was also tested with support on both sides, or another photograph was inserted, or this could be due to other reasons related to the test bench.

3 Shear Force Analysis of Reinforced Concrete Slabs

In Russian modern building codes, the method of calculating slabs for the action of shear forces appeared relatively recently and immediately caused many questions from designers and developers of programs for calculating building structures, since the results of the calculation significantly increased the required amount of shear reinforcement. The calculation of plates for the action of shear forces in accordance with clause 8.1.55 of SP 63.13330 should be performed according to the formula:

$$\frac{Q_x}{Q_{x,\mathrm{ult}}} + \frac{Q_y}{Q_{y,\mathrm{ult}}} \leq 1 \tag{1}$$

where Q_x and Q_y are shear forces acting on the lateral sides of the flat selected element; $Q_{x,\mathrm{ult}}$ and $Q_{y,\mathrm{ult}}$ are the ultimate shear forces perceived by a flat selected element and calculated by the formula:

$$Q_{\mathrm{ult}} = Q_b + Q_{sw} \tag{2}$$

where Q_b and Q_{sw} are the ultimate shear forces perceived, respectively, by concrete and shear reinforcement and determined by the formulas:

$$Q_b = 0,5 R_{bt} b h_0 \tag{3}$$

$$Q_{sw} = \frac{R_{sw} A_{sw}}{s_w} h_0 \tag{4}$$

where b and h_0 the width and effective depth of the section, respectively; A_{sw} and s_w the area of the shear reinforcement located in one normal section and the step of these rows, respectively; R_{bt} and R_{sw} tensile strength of concrete and shear reinforcement, respectively.

Previously, without taking into account this requirement, the calculation for the action of shear forces was carried out by analogy with the rods and for each direction separately, i.e., each term on the left side of expression (1) should not exceed one. Before the standard requirements of SP 63.13330.2012 came into effect in our country, millions of square meters of monolithic reinforced concrete flat slabs and foundation plates were built, which, taking into account these requirements, may no longer satisfy the ULS, and if necessary, reconstruction of buildings or structures, even if related or insignificant changes for the slabs may require strengthening to meet this requirement of the codes.

If we analyze this formula on the basis of a special case with equality $Q_{x,\text{ult}} = Q_{y,\text{ult}} = Q_{\text{ult}}$ and $Q_x = Q_y = Q$ we get:

$$\frac{Q_x}{Q_{\text{ult}}} + \frac{Q_y}{Q_{\text{ult}}} = \frac{Q_x + Q_y}{Q_{\text{ult}}} = \frac{2Q}{Q_{\text{ult}}} \leq 1 \tag{5}$$

Consequently, in this case, the ultimate shear force perceived by a flat highlighted element must be greater than twice the shear force acting on the lateral side of the element in any direction, which leads to a two-fold reserve of bearing capacity relative to consideration in separate directions. Obviously, this requirement, together with the empirical approach in determining the ultimate force for concrete according to Formula (3), which is borrowed and is only allowed in the design of rods, leads to a discrepancy between the physical nature of the shear force failure mechanism and an unjustified reserve of bearing capacity that has passed the test of time.

Another proposal for the calculation of reinforced concrete slabs of monolithic frames of multi-storey civil buildings for the action of shear forces on page 24 [3] leads to a less significant reserve of bearing capacity:

$$\left(\frac{Q_x}{Q_{x,\text{ult}}}\right)^2 + \left(\frac{Q_y}{Q_{y,\text{ult}}}\right)^2 \leq 1, \tag{6}$$

since the ratios in brackets must be less than one, and squaring them decreases the value of the sum. After similar transformations of this formula to consider the particular case $Q_{x,\text{ult}} = Q_{y,\text{ult}} = Q_{\text{ult}}$ and $Q_x = Q_y = Q$ we get:

$$\left(\frac{Q_x}{Q_{\text{ult}}}\right)^2 + \left(\frac{Q_y}{Q_{\text{ult}}}\right)^2 = \frac{Q_x^2 + Q_y^2}{Q_{\text{ult}}^2} = \frac{2Q^2}{Q_{\text{ult}}^2} \leq 1 \tag{7}$$

Therefore, in this case, the square of the ultimate shear force perceived by a flat highlighted element must be greater than twice the square of the shear force acting on the lateral side of the element in any direction, which leads to a bearing capacity margin equal to 41% (the square root of 2 is ~1.41) regarding consideration in separate directions.

In addition, it should be noted that the Formula (6) was previously used to calculate the action of the shear forces of double curvature bending (subject to bending in a plane not parallel to the axes of symmetry of the cross section) of rod reinforced concrete elements of rectangular section in accordance with clause 7.39 of SNiP II-B.1-62. And thus, when calculating one sloping section, horizontal and vertical shear bars (or stirrup segments) were taken into account to determine the ultimate shear forces along the reinforcement in two directions $Q_{sw,x}$ and $Q_{sw,y}$, y. In this case, the shear forces Q_x and Q_y act in the same section, in contrast to plates. And this nuance, obviously, contradicts the physical meaning and those test results, on the basis of which it was proposed to use this approach for calculating double curvature bent elements of rectangular section, which does not allow to unambiguously determine the possibility of using this formula for calculating reinforced concrete slabs.

In this regard, we can agree with the proposal [4] to take into account the resultant shear forces in the element and the need to consider the shear reinforcement distributed over the area of the calculated element (one amount of reinforcement for two directions), which leads to the following shape transformations of expression (1):

$$\frac{\sqrt{Q_x^2 + Q_y^2}}{Q_{\text{ult,min}}} \leq 1 \tag{8}$$

where $Q_{\text{ult,min}}$ is the minimum of the ultimate shear forces perceived by a flat selected element and calculated from the Formula (2). Or taking into account the resulting concrete in two directions and the total amount of shear reinforcement:

$$\frac{\sqrt{Q_x^2 + Q_y^2}}{\sqrt{Q_{bx}^2 + Q_{by}^2} + Q_{sw}} \leq 1 \tag{9}$$

where Q_{bx} and Q_{by} are the ultimate shear forces perceived by concrete in the x and y directions, respectively; Q_{sw} bearing capacity for forces reinforcement:

$$Q_{sw} = R_{sw} A_{sw} h_0 \tag{10}$$

where h_0 is the effective depth of section; A_{sw} is the total area of the shear reinforcement located in the element.

It should be noted that in SNiP 2.03.01-84* and Eurocode 2, there are no requirements and special calculation methods specifically for reinforced concrete slabs for the action of shear forces. And in accordance with clause 5.26 of SNiP 2.03.01-84* in solid slabs, regardless of height, it is allowed not to install shear reinforcement, while ensuring the calculation requirements according to the instructions in clause 3.32 (calculation of reinforced concrete elements without shear reinforcement for the action of a shear force to ensure strength along an sloping crack), with the exception of areas where punching shear mechanism is possible. The situation is similar in Eurocode 2.

Modern theoretical studies and proposals for improving the method of punching shear [5, 6] also continue the traditional direction and use classical assumptions when calculating for punching in the form of the absence of main sloping cracks and the possibility of uniform distribution of shear forces along the sloping face of the punching pyramid, which leads to the following contradictions:

1. When determining the bearing capacity for punching, the addition of the bearing capacity for concrete at stage I of stress–strain behavior (in the absence of cracks) and the bearing capacity for shear reinforcement at stage III of SSB (with cracks) is performed;
2. The assumption of a uniform distribution of shear stresses along the sloping faces of the punching shear pyramid has some limitations, since the relative deformations of concrete elongation are extremely small, and do not allow in practice to develop such a uniform distribution of stresses.

The proposed approach, taking into account the accepted assumptions, can be valid only for certain cases, for example, for concrete structures, where bending stresses, due to their smallness, can be neglected, or in prestressed reinforced concrete structures in the absence of normal and sloping cracks.

4 Punching Experimental Analysis of Flat Slab

An analysis of the normative methods of punching shear design in accordance with SNiP 2.03.01-84*, SP 63.13330.2012 and Eurocode 2, taking into account the experimental data obtained, and a proposal for the development of the method are published in article [7]. According to the test results, it was found that the sloping angle of the punching shear pyramid (main sloping crack) was about 20° and normal cracks on the lateral surfaces (in contrast to ΠI-5a shown in Fig. 1) did not form, due to the impossibility of bending the element directly on the support contour.

For testing [7], a support contour with dimensions in plan of 2.16 × 2.16 × 0.24 (h) m was made from a paired channel No. 24 of a box-shaped cross section. The support contour was installed on 8 racks 1.2 m high, resting on the load-bearing floor. The slab prototype was installed on a sand-cement mortar with a support length of 100 mm along the supporting contour. The loading was carried out through a load plate by 4 hydraulic power exciters from a common collector, capable of creating a force of 1000 kN each. The total mass of the tooling applied before the start of the experiment was 80 kN. Further, the load applied to the slab fragment through the column changed stepwise, increasing by 200 kN with an interval of 20 min, immediately until the moment of destruction.

In the manufacture of a slab fragment, heavy concrete of class B30 and reinforcement of class A500C 18Ø12 mm with a step of 100 mm in two directions in the lower zone at a distance from the center of 1 row of reinforcement to the edge of the slab equal to 25 mm were used. The overall dimensions of the slab fragment are 2.0 × 2.0 × 0.24 (h) m. The column head was made in the form of a cube with a side size of 0.5 m and structural reinforcement of longitudinal (4Ø16 mm) and shear reinforcement (5Ø8 mm with step 100 mm).

It is interesting to note the general regularity according to the test results of beams and slabs without shear reinforcement in [2, 7], associated with the formation of main sloping cracks from supports on stretched faces to column face on compressed faces of prototype with a slope angle significantly less than 45°. In addition, an important not previously noted nuance according to the results of observation of the destruction of the sample in [7] is Fig. 3—this is the exit of the main sloping cracks on a stretched horizontal surface at opposite supports in the direction of longitudinal reinforcement located with a smaller concrete cover, and the absence of similar cracks of such an development on the stretched surface in the perpendicular direction, which is associated with the operation of the hydraulic loading system and almost instantaneous zeroing pressure (external load) upon destruction of the sample.

In this case, after the transition to the plastic stage of the structure's operation, when most of the bars of two directions cease to resist external load and a significant increase in deflections is recorded, the factor of reinforcement placement in thickness begins to influence—the closer the longitudinal reinforcement is to the tensioned surface of the slab, the more it bends near the support and in the event of an inflection point in the reinforcement or a certain angle of rotation of the section, it begins to provoke concrete cleavage due to the occurrence of reactive pressure of the reinforcement and the exit of the main sloping cracks on the tensioned surface of the prototype, which, together with the destruction of the compressed zone of concrete around the head of the column (load area) by cut (not necessarily in the plane of the normal section) leads to the formation of a punching shear pyramid.

It should be noted that most of the tests of various conventional prototypes for punching without shear reinforcement were carried out with an almost equal ratio of the sloping section span (the length of the horizontal projection of an sloping crack) to the height of the elements, which led to the formation of a punching shear pyramid at an angle close to 45°. This is due to the limitation of the dimensions in terms of the tested images by the capabilities of laboratory equipment (4 rack presses). But if it was possible to test samples of large dimensions in plan [2, 7], then the angle of sloping reached 20°. It is also obvious that in the absence of bending of the specimens (at $h_0/c > 2$) or sufficient resistance of concrete to tensile stresses without cracking and the ratio of the height of the elements to the sloping section span of more than one, the slope of the punching shear pyramid will exceed 45° and tend to 90° in the

Fig. 3 Prototype in [7] after punching failure. Tensioned surface

limit:

$$\lim_{h_0/c \to \infty} \varphi = 90^\circ \tag{11}$$

where c and φ are the sloping section span and the angle of sloping, respectively.

At the same time, to obtain results close to the experimental data, it is necessary, instead of the tensile resistance R_{bt} of concrete, to use the concrete shear resistance R_{bs}, which reaches, according to various estimates, up to $2.5R_{bt}$ without taking into account the lateral compression [8].

Another aspect is related to the options for loading reinforced concrete slabs and their support on vertical supporting structures. Obviously, in flat slabs with support on columns, pylons or walls under the action of loads uniformly distributed over the area (or close to them) and sufficient bearing capacity for the action of shear forces only on concrete, there is no need to calculate the slabs for the shear force outside the support zones. And vice versa, in the case of a concentrated load acting on the overlap and a sufficient bearing capacity with respect to the shear force only on concrete at the place of its application, there is no need to check the strength on the supports. Similar statements apply to foundation slabs as well. Despite all its obviousness, this of course requires rigorous proof. Using the example of a flat slab, uniformly distributed load and the same dimensions of elements in the plan:

1. Given:

   q—uniformly distributed load;
   A_i—area of the i-th slab element;
   R_j—reaction in the j-th vertical support;
   n—number of elements of flat slabs;
   m—number of supports.

2. Let's compose the equation of equilibrium of the acting support reactions and vertical forces on the vertical axis Z:

$$\sum Q_z = \sum_{i=1}^{n} qA_i + \sum_{j=1}^{m} R_j = 0 \quad (12)$$

3. Since in practice, the number of flat slab elements is always significantly greater than the number of supports:

$$n \gg m, \quad (13)$$

   then, taking into account equality (12):

$$qA_i < R_j, \quad (14)$$

   then, according to (14), while ensuring the bearing capacity for the action of shear forces on concrete on the support, the strength of any individual slab element will be ensured, which was required to be proved.

5 Punching Catastrophic Failure Analysis

It should also be noted that in real monolithic reinforced concrete flat slabs under the action of uniformly distributed loads, the main sloping cracks can form at an angle close to 45°. This is confirmed by the specific example shown in Fig. 4, catastrophic failure of the flat slab of the stylobate part of the building at the interface with the column at the facility in Moscow near the Paveletsky railway station in 2009. Then, at the stage of developing the detailed design documentation, the new design organization made changes to the initial project, which passed the examination, in

Fig. 4 Flat slab after punching failure

the form of excluding capitals and reducing the cross section of columns, which led to the implementation of a punching mechanism.

This aspect was most likely known to the authors of the method for calculating reinforced concrete slabs for punching and therefore, taking into account other experimental studies, served as the basis for using the corresponding sloping angle of the pyramid in the codes. But nevertheless, such an approach, taking into account a wide variety of design solutions, is not universal, has a number of the above disadvantages and requires further development.

In the following publications, on the basis of experimental studies [2, 7, 8] and proposals for improving the method of calculating slabs for punching shear in [7], a comparative analysis of the calculations of the bearing capacity for the action of shear forces and punching shear will be carried out to develop a unified and universal approach to the calculation of reinforced concrete slabs.

6 Conclusions

1. Based on the analysis of the experimental data presented, it can be stated that the failure of plates under the action of shear forces and from punching has a related nature and a similar mechanism due to the formation and development of main sloping cracks.

2. The current normative calculation method in accordance with the requirements of clause 8.1.55 of SP 63.13330.2018 for the action of shear forces has a significant reserve for the bearing capacity and may lead to the need to strengthen reinforced concrete slabs not affected by the reconstruction of buildings and structures.
3. Against this background and the current volume of construction with the use of monolithic reinforced concrete, the creation of a unified and universal method for calculating reinforced concrete slabs for the action of shear forces and punching is an urgent task.

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